

Problems In Elementary Number Theory Problem Solving

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Elements of Number Theory - I. M. Vinogradov 2016-01-14

Clear, detailed exposition that can be understood by readers with no background in advanced mathematics. More than 200 problems and full solutions, plus 100 numerical exercises. 1949 edition.

Solved and Unsolved Problems in Number Theory - Daniel Shanks

2020-12-29

Much of elementary number theory arose out of the investigation of three problems; that of perfect numbers, that of periodic decimals, and that of Pythagorean numbers. We have accordingly organized the book into three long chapters. The result of such an organization is that motivation is stressed to a rather unusual degree. Theorems arise in response to previously posed problems, and their proof is sometimes delayed until an appropriate analysis can be developed. These theorems, then, or most of them, are "solved problems." Historical discussion is, of course, natural in such a presentation. However, our primary interest is in the theorems, and their logical interrelations, and not in the history per se. The aspect of the historical approach which mainly concerns us is the determination of the problems which suggested the theorems, and the study of which provided the concepts and the techniques which were later used in their proof. In most number theory books residue classes are introduced prior to Fermat's Theorem and the Reciprocity Law. But

this is not at all the correct historical order. We have here restored these topics to their historical order, and it seems to us that this restoration presents matters in a more natural light. The "unsolved problems" are the conjectures and the open questions- we distinguish these two categories-and these problems are treated more fully than is usually the case. The conjectures, like the theorems, are introduced at the point at which they arise naturally, are numbered and stated formally. Their significance, their interrelations, and the heuristic evidence supporting them are often discussed. It is well-known that some unsolved problems, such as Fermat's Last Theorem and Riemann's Hypothesis, have been enormously fruitful in suggesting new mathematical fields, and for this reason alone it is not desirable to dismiss conjectures without an adequate discussion. Further, number theory is very much a live subject, and it seems desirable to emphasize this.

Elementary Number Theory - David M. Burton 2002

This text provides a simple account of classical number theory, as well as some of the historical background in which the subject evolved. It is intended for use in a one-semester, undergraduate number theory course taken primarily by mathematics majors and students preparing to be secondary school teachers. Although the text was written with this readership in mind, very few formal prerequisites are required. Much of

the text can be read by students with a sound background in high school mathematics.

Solved and Unsolved Problems in Number Theory - Daniel Shanks 2001

The investigation of three problems, perfect numbers, periodic decimals, and Pythagorean numbers, has given rise to much of elementary number theory. In this book, Daniel Shanks, past editor of *Mathematics of Computation*, shows how each result leads to further results and conjectures. The outcome is a most exciting and unusual treatment. This edition contains a new chapter presenting research done between 1962 and 1978, emphasizing results that were achieved with the help of computers.

100 Great Problems of Elementary Mathematics - Heinrich Dörrie 2013-04-09

Problems that beset Archimedes, Newton, Euler, Cauchy, Gauss, Monge, Steiner, and other great mathematical minds. Features squaring the circle, pi, and similar problems. No advanced math is required. Includes 100 problems with proofs.

The Higher Arithmetic - H. Davenport 2008-10-23

The theory of numbers is generally considered to be the 'purest' branch of pure mathematics and demands exactness of thought and exposition from its devotees. It is also one of the most highly active and engaging areas of mathematics. Now into its eighth edition *The Higher Arithmetic* introduces the concepts and theorems of number theory in a way that does not require the reader to have an in-depth knowledge of the theory of numbers but also touches upon matters of deep mathematical significance. Since earlier editions, additional material written by J. H. Davenport has been added, on topics such as Wiles' proof of Fermat's Last Theorem, computers and number theory, and primality testing. Written to be accessible to the general reader, with only high school mathematics as prerequisite, this classic book is also ideal for undergraduate courses on number theory, and covers all the necessary material clearly and succinctly.

1001 Problems in Classical Number Theory - Armel Mercier 2007

Elementary Number Theory - Gareth A. Jones 2012-12-06

An undergraduate-level introduction to number theory, with the emphasis on fully explained proofs and examples. Exercises, together with their solutions are integrated into the text, and the first few chapters assume only basic school algebra. Elementary ideas about groups and rings are then used to study groups of units, quadratic residues and arithmetic functions with applications to enumeration and cryptography. The final part, suitable for third-year students, uses ideas from algebra, analysis, calculus and geometry to study Dirichlet series and sums of squares. In particular, the last chapter gives a concise account of Fermat's Last Theorem, from its origin in the ancient Babylonian and Greek study of Pythagorean triples to its recent proof by Andrew Wiles.

Solving Mathematical Problems - Terence Tao 2006-07-28

Authored by a leading name in mathematics, this engaging and clearly presented text leads the reader through the tactics involved in solving mathematical problems at the Mathematical Olympiad level. With numerous exercises and assuming only basic mathematics, this text is ideal for students of 14 years and above in pure mathematics.

Not Always Buried Deep - Paul Pollack 2009-10-14

Number theory is one of the few areas of mathematics where problems of substantial interest can be fully described to someone with minimal mathematical background. Solving such problems sometimes requires difficult and deep methods. But this is not a universal phenomenon; many engaging problems can be successfully attacked with little more than one's mathematical bare hands. In this case one says that the problem can be solved in an elementary way. Such elementary methods and the problems to which they apply are the subject of this book. *Not Always Buried Deep* is designed to be read and enjoyed by those who wish to explore elementary methods in modern number theory. The heart of the book is a thorough introduction to elementary prime number theory, including Dirichlet's theorem on primes in arithmetic progressions, the Brun sieve, and the Erdos-Selberg proof of the prime number theorem. Rather than trying to present a comprehensive treatise, Pollack focuses

on topics that are particularly attractive and accessible. Other topics covered include Gauss's theory of cyclotomy and its applications to rational reciprocity laws, Hilbert's solution to Waring's problem, and modern work on perfect numbers. The nature of the material means that little is required in terms of prerequisites: The reader is expected to have prior familiarity with number theory at the level of an undergraduate course and a first course in modern algebra (covering groups, rings, and fields). The exposition is complemented by over 200 exercises and 400 references.

Number Theory - Titu Andreescu 2009-06-12

This introductory textbook takes a problem-solving approach to number theory, situating each concept within the framework of an example or a problem for solving. Starting with the essentials, the text covers divisibility, unique factorization, modular arithmetic and the Chinese Remainder Theorem, Diophantine equations, binomial coefficients, Fermat and Mersenne primes and other special numbers, and special sequences. Included are sections on mathematical induction and the pigeonhole principle, as well as a discussion of other number systems. By emphasizing examples and applications the authors motivate and engage readers.

Hilbert's Tenth Problem: An Introduction to Logic, Number Theory, and Computability - M. Ram Murty 2019-05-09

Hilbert's tenth problem is one of 23 problems proposed by David Hilbert in 1900 at the International Congress of Mathematicians in Paris. These problems gave focus for the exponential development of mathematical thought over the following century. The tenth problem asked for a general algorithm to determine if a given Diophantine equation has a solution in integers. It was finally resolved in a series of papers written by Julia Robinson, Martin Davis, Hilary Putnam, and finally Yuri Matiyasevich in 1970. They showed that no such algorithm exists. This book is an exposition of this remarkable achievement. Often, the solution to a famous problem involves formidable background. Surprisingly, the solution of Hilbert's tenth problem does not. What is needed is only some elementary number theory and rudimentary logic. In this book, the

authors present the complete proof along with the romantic history that goes with it. Along the way, the reader is introduced to Cantor's transfinite numbers, axiomatic set theory, Turing machines, and Gödel's incompleteness theorems. Copious exercises are included at the end of each chapter to guide the student gently on this ascent. For the advanced student, the final chapter highlights recent developments and suggests future directions. The book is suitable for undergraduates and graduate students. It is essentially self-contained.

Problem Based Journey From Elementary Number Theory To An Introduction To Matrix Theory, A: The President Problems - Abraham Berman 2021-10-18

The book is based on lecture notes of a course 'from elementary number theory to an introduction to matrix theory' given at the Technion to gifted high school students. It is problem based, and covers topics in undergraduate mathematics that can be introduced in high school through solving challenging problems. These topics include Number theory, Set Theory, Group Theory, Matrix Theory, and applications to cryptography and search engines.

Number Theory - Kuldeep Singh 2020-10-08

Number theory is one of the oldest branches of mathematics that is primarily concerned with positive integers. While it has long been studied for its beauty and elegance as a branch of pure mathematics, it has seen a resurgence in recent years with the advent of the digital world for its modern applications in both computer science and cryptography. *Number Theory: Step by Step* is an undergraduate-level introduction to number theory that assumes no prior knowledge, but works to gradually increase the reader's confidence and ability to tackle more difficult material. The strength of the text is in its large number of examples and the step-by-step explanation of each topic as it is introduced to help aid understanding the abstract mathematics of number theory. It is compiled in such a way that allows self-study, with explicit solutions to all the set of problems freely available online via the companion website. Punctuating the text are short and engaging historical profiles that add context for the topics covered and provide a

dynamic background for the subject matter.

The USSR Olympiad Problem Book - D. O. Shklarsky 2013-04-15

Over 300 challenging problems in algebra, arithmetic, elementary number theory and trigonometry, selected from Mathematical Olympiads held at Moscow University. Only high school math needed. Includes complete solutions. Features 27 black-and-white illustrations. 1962 edition.

Number Theory for Computing - Song Y. Yan 2002-04-24

This book provides a good introduction to the classical elementary number theory and the modern algorithmic number theory, and their applications in computing and information technology, including computer systems design, cryptography and network security. In this second edition proofs of many theorems have been provided, further additions and corrections were made.

Discrete Mathematics and Its Applications - Kenneth H. Rosen 2007

The companion Web site -- To the student -- The foundations : logic, sets, and functions -- The fundamentals : algorithms, the integers, and matrices -- Mathematical reasoning -- Counting -- Advanced counting techniques -- Relations -- Graphs -- Trees -- Boolean algebra -- Modeling computation

Lectures on Number Theory - Peter Gustav Lejeune Dirichlet 1999

Lectures on Number Theory is the first of its kind on the subject matter. It covers most of the topics that are standard in a modern first course on number theory, but also includes Dirichlet's famous results on class numbers and primes in arithmetic progressions.

Putnam and Beyond - Răzvan Gelca 2017-09-19

This book takes the reader on a journey through the world of college mathematics, focusing on some of the most important concepts and results in the theories of polynomials, linear algebra, real analysis, differential equations, coordinate geometry, trigonometry, elementary number theory, combinatorics, and probability. Preliminary material provides an overview of common methods of proof: argument by contradiction, mathematical induction, pigeonhole principle, ordered sets, and invariants. Each chapter systematically presents a single

subject within which problems are clustered in each section according to the specific topic. The exposition is driven by nearly 1300 problems and examples chosen from numerous sources from around the world; many original contributions come from the authors. The source, author, and historical background are cited whenever possible. Complete solutions to all problems are given at the end of the book. This second edition includes new sections on quadratic polynomials, curves in the plane, quadratic fields, combinatorics of numbers, and graph theory, and added problems or theoretical expansion of sections on polynomials, matrices, abstract algebra, limits of sequences and functions, derivatives and their applications, Stokes' theorem, analytical geometry, combinatorial geometry, and counting strategies. Using the W.L. Putnam Mathematical Competition for undergraduates as an inspiring symbol to build an appropriate math background for graduate studies in pure or applied mathematics, the reader is eased into transitioning from problem-solving at the high school level to the university and beyond, that is, to mathematical research. This work may be used as a study guide for the Putnam exam, as a text for many different problem-solving courses, and as a source of problems for standard courses in undergraduate mathematics. Putnam and Beyond is organized for independent study by undergraduate and graduate students, as well as teachers and researchers in the physical sciences who wish to expand their mathematical horizons.

Problems of Number Theory in Mathematical Competitions - Hong-Bing Yu 2010

Number theory is an important research field of mathematics. In mathematical competitions, problems of elementary number theory occur frequently. These problems use little knowledge and have many variations. They are flexible and diverse. In this book, the author introduces some basic concepts and methods in elementary number theory via problems in mathematical competitions. Readers are encouraged to try to solve the problems by themselves before they read the given solutions of examples. Only in this way can they truly appreciate the tricks of problem-solving.

Unsolved Problems in Number Theory - Richard Guy 2013-06-29
Second edition sold 2241 copies in N.A. and 1600 ROW. New edition contains 50 percent new material.

Problems and Proofs in Numbers and Algebra - Richard S. Millman
2015-03-09

Focusing on an approach of solving rigorous problems and learning how to prove, this volume is concentrated on two specific content themes, elementary number theory and algebraic polynomials. The benefit to readers who are moving from calculus to more abstract mathematics is to acquire the ability to understand proofs through use of the book and the multitude of proofs and problems that will be covered throughout. This book is meant to be a transitional precursor to more complex topics in analysis, advanced number theory, and abstract algebra. To achieve the goal of conceptual understanding, a large number of problems and examples will be interspersed through every chapter. The problems are always presented in a multi-step and often very challenging, requiring the reader to think about proofs, counter-examples, and conjectures. Beyond the undergraduate mathematics student audience, the text can also offer a rigorous treatment of mathematics content (numbers and algebra) for high-achieving high school students. Furthermore, prospective teachers will add to the breadth of the audience as math education majors, will understand more thoroughly methods of proof, and will add to the depth of their mathematical knowledge. In the past, PNA has been taught in a "problem solving in middle school" course (twice), to a quite advanced high school students course (three semesters), and three times as a secondary resource for a course for future high school teachers. PNA is suitable for secondary math teachers who look for material to encourage and motivate more high achieving students.

Methods of Solving Number Theory Problems - Ellina Grigorieva
2018-07-06

Through its engaging and unusual problems, this book demonstrates methods of reasoning necessary for learning number theory. Every technique is followed by problems (as well as detailed hints and

solutions) that apply theorems immediately, so readers can solve a variety of abstract problems in a systematic, creative manner. New solutions often require the ingenious use of earlier mathematical concepts - not the memorization of formulas and facts. Questions also often permit experimental numeric validation or visual interpretation to encourage the combined use of deductive and intuitive thinking. The first chapter starts with simple topics like even and odd numbers, divisibility, and prime numbers and helps the reader to solve quite complex, Olympiad-type problems right away. It also covers properties of the perfect, amicable, and figurate numbers and introduces congruence. The next chapter begins with the Euclidean algorithm, explores the representations of integer numbers in different bases, and examines continued fractions, quadratic irrationalities, and the Lagrange Theorem. The last section of Chapter Two is an exploration of different methods of proofs. The third chapter is dedicated to solving Diophantine linear and nonlinear equations and includes different methods of solving Fermat's (Pell's) equations. It also covers Fermat's factorization techniques and methods of solving challenging problems involving exponent and factorials. Chapter Four reviews the Pythagorean triple and quadruple and emphasizes their connection with geometry, trigonometry, algebraic geometry, and stereographic projection. A special case of Waring's problem as a representation of a number by the sum of the squares or cubes of other numbers is covered, as well as quadratic residuals, Legendre and Jacobi symbols, and interesting word problems related to the properties of numbers. Appendices provide a historic overview of number theory and its main developments from the ancient cultures in Greece, Babylon, and Egypt to the modern day. Drawing from cases collected by an accomplished female mathematician, *Methods in Solving Number Theory Problems* is designed as a self-study guide or supplementary textbook for a one-semester course in introductory number theory. It can also be used to prepare for mathematical Olympiads. Elementary algebra, arithmetic and some calculus knowledge are the only prerequisites. Number theory gives precise proofs and theorems of an irreproachable rigor and sharpens analytical thinking,

which makes this book perfect for anyone looking to build their mathematical confidence.

Equations and Inequalities - Jiri Herman 2012-12-06

A look at solving problems in three areas of classical elementary mathematics: equations and systems of equations of various kinds, algebraic inequalities, and elementary number theory, in particular divisibility and diophantine equations. In each topic, brief theoretical discussions are followed by carefully worked out examples of increasing difficulty, and by exercises which range from routine to rather more challenging problems. While it emphasizes some methods that are not usually covered in beginning university courses, the book nevertheless teaches techniques and skills which are useful beyond the specific topics covered here. With approximately 330 examples and 760 exercises.

250 Problems in Elementary Number Theory - Waclaw Sierpinski 1970

Elementary Number Theory: Primes, Congruences, and Secrets - William Stein 2009-01-08

Classical number theory and elliptic curves are examined in this textbook, which moves on from elementary topics such as primes, continued fractions, and quadratic forms, to elliptic curves and their applications to algorithmic and number theory problems.

Number Theory - Titu Andreescu 2017-07-15

Challenge your problem-solving aptitude in number theory with powerful problems that have concrete examples which reflect the potential and impact of theoretical results. Each chapter focuses on a fundamental concept or result, reinforced by each of the subsections, with scores of challenging problems that allow you to comprehend number theory like never before. All students and coaches wishing to excel in math competitions will benefit from this book as will mathematicians and adults who enjoy interesting mathematics.

Problem-Solving and Selected Topics in Number Theory - Michael Th. Rassias 2010-11-16

The book provides a self-contained introduction to classical Number

Theory. All the proofs of the individual theorems and the solutions of the exercises are being presented step by step. Some historical remarks are also presented. The book will be directed to advanced undergraduate, beginning graduate students as well as to students who prepare for mathematical competitions (ex. Mathematical Olympiads and Putnam Mathematical competition).

Problems in Algebraic Number Theory - M. Ram Murty 2005

It has become clear that problem solving plays an extremely important role in mathematical research. This book is a collection of about 500 problems in algebraic number theory. They are systematically arranged to reveal the evolution of concepts and ideas of the subject. For this new edition the authors have added a new chapter and revised several sections.

Steps into Analytic Number Theory - Paul Pollack 2021-02-08

This problem book gathers together 15 problem sets on analytic number theory that can be profitably approached by anyone from advanced high school students to those pursuing graduate studies. It emerged from a 5-week course taught by the first author as part of the 2019 Ross/Asia Mathematics Program held from July 7 to August 9 in Zhenjiang, China. While it is recommended that the reader has a solid background in mathematical problem solving (as from training for mathematical contests), no possession of advanced subject-matter knowledge is assumed. Most of the solutions require nothing more than elementary number theory and a good grasp of calculus. Problems touch at key topics like the value-distribution of arithmetic functions, the distribution of prime numbers, the distribution of squares and nonsquares modulo a prime number, Dirichlet's theorem on primes in arithmetic progressions, and more. This book is suitable for any student with a special interest in developing problem-solving skills in analytic number theory. It will be an invaluable aid to lecturers and students as a supplementary text for introductory Analytic Number Theory courses at both the undergraduate and graduate level.

Problem Solving Strategies for Elementary-School Math - Kiana Avestimehr 2020-06-24

This book teaches 7 basic problem solving strategies that can be used by elementary students to overcome the challenge of how to start thinking about a math problem. It contains more than 100 challenging problems that are suitable for elementary-school students, along with their step-by-step solution to help the reader master these strategies.

Elementary Number Theory - James S. Kraft 2014-11-24

Elementary Number Theory takes an accessible approach to teaching students about the role of number theory in pure mathematics and its important applications to cryptography and other areas. The first chapter of the book explains how to do proofs and includes a brief discussion of lemmas, propositions, theorems, and corollaries. The core of the text covers linear Diophantine equations; unique factorization; congruences; Fermat's, Euler's, and Wilson's theorems; order and primitive roots; and quadratic reciprocity. The authors also discuss numerous cryptographic topics, such as RSA and discrete logarithms, along with recent developments. The book offers many pedagogical features. The "check your understanding" problems scattered throughout the chapters assess whether students have learned essential information. At the end of every chapter, exercises reinforce an understanding of the material. Other exercises introduce new and interesting ideas while computer exercises reflect the kinds of explorations that number theorists often carry out in their research.

Problem-Solving Strategies - Arthur Engel 2008-01-19

A unique collection of competition problems from over twenty major national and international mathematical competitions for high school students. Written for trainers and participants of contests of all levels up to the highest level, this will appeal to high school teachers conducting a mathematics club who need a range of simple to complex problems and to those instructors wishing to pose a "problem of the week", thus bringing a creative atmosphere into the classrooms. Equally, this is a must-have for individuals interested in solving difficult and challenging problems. Each chapter starts with typical examples illustrating the central concepts and is followed by a number of carefully selected problems and their solutions. Most of the solutions are complete, but

some merely point to the road leading to the final solution. In addition to being a valuable resource of mathematical problems and solution strategies, this is the most complete training book on the market.

Elementary Number Theory: Primes, Congruences, and Secrets - William Stein 2008-10-28

This is a book about prime numbers, congruences, secret messages, and elliptic curves that you can read cover to cover. It grew out of undergraduate courses that the author taught at Harvard, UC San Diego, and the University of Washington. The systematic study of number theory was initiated around 300B. C. when Euclid proved that there are infinitely many prime numbers, and also cleverly deduced the fundamental theorem of arithmetic, which asserts that every positive integer factors uniquely as a product of primes. Over a thousand years later (around 972A. D.) Arab mathematicians formulated the congruent number problem that asks for a way to decide whether or not a given positive integer n is the area of a right triangle, all three of whose sides are rational numbers. Then another thousand years later (in 1976), Diffie and Hellman introduced the first ever public-key cryptosystem, which enabled two people to communicate secretly over a public communications channel with no predetermined secret; this invention and the ones that followed it revolutionized the world of digital communication. In the 1980s and 1990s, elliptic curves revolutionized number theory, providing striking new insights into the congruent number problem, primality testing, public-key cryptography, attacks on public-key systems, and playing a central role in Andrew Wiles' resolution of Fermat's Last Theorem.

Elementary Number Theory with Applications - Thomas Koshy 2002

The advent of modern technology has brought a new dimension to the power of number theory: constant practical use. Once considered the purest of pure mathematics, it is used increasingly now in the rapid development of technology in a number of areas, such as art, coding theory, cryptology, computer science, and other necessities of modern life. Elementary Number Theory with Applications is the fruit of years of dreams and the author's fascination with the subject, encapsulating the

beauty, elegance, historical development, and opportunities provided for experimentation and application. This is the only number theory book to show how modular systems can be employed to create beautiful designs, thus linking number theory with both geometry and art. It is also the only number theory book to deal with bar codes, Zip codes, International Standard Book Numbers (ISBN), and European Article Numbers (EAN). Emphasis is on problem-solving strategies (doing experiments, collecting and organizing data, recognizing patterns, and making conjectures). Each section provides a wealth of carefully prepared, well-graded examples and exercises to enhance the readers' understanding and problem-solving skills. This is the only number theory book to: Show how modular systems can be employed to create beautiful designs, thus linking number theory with both geometry and art Deal with bar codes, Zip codes, International Standard Book Numbers (ISBN), and European Article Numbers (EAN) Emphasize problem-solving strategies (doing experiments, collecting and organizing data, recognizing patterns, and making conjectures) Provide a wealth of carefully prepared, well-graded examples and exercises to enhance the readers' understanding and problem-solving skills

Problems in Algebraic Number Theory - M. Ram Murty 2006-03-30

The problems are systematically arranged to reveal the evolution of concepts and ideas of the subject Includes various levels of problems - some are easy and straightforward, while others are more challenging All problems are elegantly solved

Introduction to Number Theory - Mathew Crawford 2008

Introduction to Matrix Theory - Arindama Singh 2021-08-16

This book is designed to serve as a textbook for courses offered to undergraduate and postgraduate students enrolled in Mathematics. Using elementary row operations and Gram-Schmidt orthogonalization as basic tools the text develops characterization of equivalence and similarity, and various factorizations such as rank factorization, QR-factorization, Schurtriangularization, Diagonalization of normal matrices, Jordan decomposition, singular value decomposition, and polar decomposition. Along with Gauss-Jordan elimination for linear systems, it also discusses best approximations and least-squares solutions. The book includes norms on matrices as a means to deal with iterative solutions of linear systems and exponential of a matrix. The topics in the book are dealt with in a lively manner. Each section of the book has exercises to reinforce the concepts, and problems have been added at the end of each chapter. Most of these problems are theoretical, and they do not fit into the running text linearly. The detailed coverage and pedagogical tools make this an ideal textbook for students and researchers enrolled in senior undergraduate and beginning postgraduate mathematics courses.

The Theory of Numbers - Andrew Adler 1995

Elementary Number Theory - Underwood Dudley 1978

"With almost a thousand imaginative exercises and problems, this book stimulates curiosity about numbers and their properties."